Full-wave simulation of focusing light through scattering layers using the T-matrix method

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Abstract

We couple the T-matrix method with a discrete particle representation of turbid media to simulate the focusing light through a highly scattering titanium dioxide phantom. We have used our method to simulate wavefront shaping with full phase modulation using a stepwise sequential algorithm, and have generated multiple foci and evaluated their enhancement against theory. Our computationally efficient yet physically realistic technique allows researchers to resolve both amplitude and phase information at arbitrary locations inside and through bespoke scattering media.

Optical wavefront shaping (WFS) is a method of controlling the propagation of light through scattering media by spatially modulating the incident wavefront [1]. This allows light to constructively interfere to produce an optical focus at depth [1] or to image through turbid media and around corners [2]. Experimental investigation of WFS can be complemented by computational approaches. This is because computational methods can easily resolve both amplitude and phase information, can directly evaluate the field inside a given medium, and are able to comprehensively and dynamically control the domain with respect to optical properties or geometry. However, current simulation methods are either too computationally challenging to model volumes of at least a transport mean free path (where light propagation becomes diffuse and WFS has the greatest potential benefit), or too incomplete to model the underlying deterministic scattering and interference processes that characterise physically realistic light transport.

To address the challenge of designing a computationally efficient yet physically rigorous simulation of WFS we have coupled the T-matrix method with a discrete particle model of turbid media. This full-wave method directly solves Maxwell's equations (and as such can accurately simulate the physics of light scattering) but does not require a computationally expensive subwavelength discretisation of the simulation domain. Instead, the T-matrix method works by propagating light through a medium of scattering particles (most commonly spheres) such that the total field is a superposition of the scattered fields associated with each sphere [3]. This field is calculated by solving a linear system governing how each sphere interacts with every other sphere. The angular spectrum method is then used to simulate the complex beams found in WFS by decomposing the incident light into a spectrum of plane waves, which can be simulated sequentially [4].

To demonstrate the technique, the seminal Vellekoop and Mosk WFS experiment [1] is replicated *in silico* by simulating the generation of an optical focus through a titanium dioxide (TiO_2) scattering layer using a full-phase stepwise sequential algorithm. A 30x30x10μm^3 domain was constructed comprised of TiO_2 scattering spheres with a radius of 1μm and refractive index of 2.6 at a concentration of 0.26 by volume. The background refractive index was set to 1.33 and 441 different plane waves (λ=633nm) were simulated propagating through the medium using the CELES software [5], with the polar and azimuthal angles of the incident waves varying from -10 to 10°. Mie theory was used to design this domain such that it has a transport mean free path of $\sim$5μm, which ensures that there is no correlation between the input and output fields.

Figure 1 demonstrates how simulating a plane wave produces a speckle pattern on a 20x20μm^2 plane 20μm behind medium - there are no correlations in the field and the mean intensity is lower as the light has been heavily scattered. An angular spectrum decomposition is used as a spatial light modulator (SLM) analogue - speckle patterns are generated sequentially for 441 different planes waves incident at a range of angles. A stepwise sequential algorithm iterates through these simulated fields, modulating the phase between 0 and 2π in increments of $1/4\pi$ and records the change in intensity in a target region relative to a reference plane wave. Calculating the superposition of all the modulated waves that maximise the intensity in the target region produces a field with a strong optical focus, as seen in Figure 1. Computational methods are advantageous in that they are able to evaluate the field at arbitrary locations, both inside and outside the medium. This is demonstrated in Figure 1 in which field transversely bisecting the scattering medium is plotted. From this perspective it is clear to see that while the plane wave is heavily scattered, the shaped light interferes to produce an optical focus. The ability to evaluate internal fields has implication for researchers looking to generate internal foci without the need for a feedback mechanism [6].

Like Vellekoop and Mosk, we evaluated how the intensity of our focus scales with the number of incident plane waves (analogous to SLM elements) [1]. The comparison of these computationally derived enhancements (defined as the ratio between the intensity of the focus and the average intensity of the unoptimised field) to the theoretical values [1] are plotted in Figure 2. There is a strong linear correlation between the enhancement and the number of elements, and good agreement between the computational and theoretical values. These simulated fields and the foundational discrete particle domains have both been validated through comparison with the pseudospectral time-domain method and inverse adding-doubling, although these results are not shown for brevity.

As this technique is able to optimise the incident phase map to attempt to generate any arbitrary target field it is possible to simulate the generation of multiple foci simultaneously. As compared to before, the stepwise sequential algorithm is now optimising the incident field to maximise the intensities at five separate points simultaneously, which produces the phase mask and field seen in Figure 3. The proposed technique can also simulate amplitude based modulation using a variety of shaping algorithms, and can directly evaluate the transmission matrix of a scattering medium.

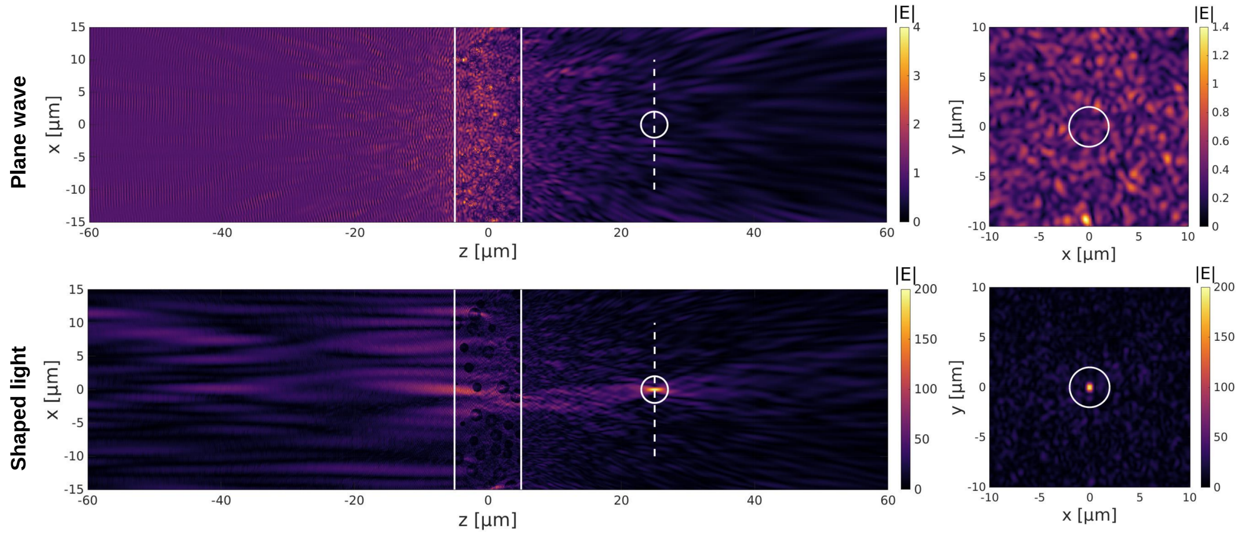

Fig. 1: The TiO_2 domain (delineated by the solid white lines) scatters an incident plane wave, but a properly shaped wavefront can generate a focus in the target region (highlighted by the white circle). The generated optical focus can be clearly seen by imaging the xy plane across the target region, depicted by the dashed lines.

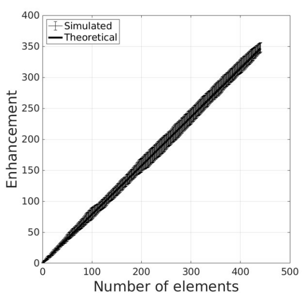

Fig. 2: Correlation between simulated and theoretical enhancements as a function of the number of elements used to generate a focus. We find that our results match theory.

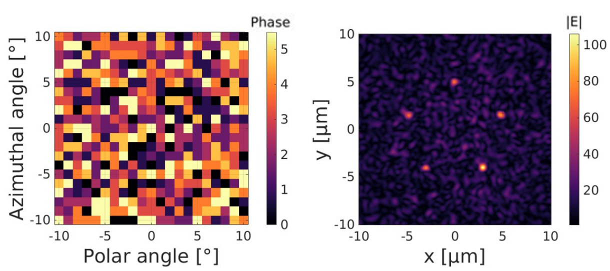

Fig. 3: Simulating the generation of multiple foci through our TiO_2 phantom using the stepwise sequential algorithm of to produce a phase mask.

Coupling the full-wave T-matrix method with a discrete particle representation of turbid media produced a physically realistic and computationally efficient method of simulating light propagation through scattering domains; an angular spectrum decomposition can then be used as a SLM analogue to simulate WFS and the focusing of light through this domain. As previously mentioned, rigorous computational investigation of WFS is important both due to the ability to fully control the simulation domain and to evaluate the field at arbitrary locations internally and externally. An example of this would be Yang *et al.* simulating the focusing of light inside dynamic tissue using a phase screen approach [7]. While useful, such approaches confine scattering to discrete planes, and allow for no back-scattering or ballistic light. In contrast, our proposed full-wave method fully models the physics of scattering, which is consistent with the strong agreement between simulated and theoretical enhancement shown in Figure 2. At present, we are using the proposed computational approach to investigate various WFS phenomena. For example: How significantly does each SLM element contribute to generating a focus? How does enhancement scale when generating multiple foci? How does enhancement scale as a function of target area size? How do various shaping algorithms compare and how generalisable are deep learning based methods? How do the optical properties of a domain affect WFS? We have also used this method to investigate other coherent phenomena such as the angular memory effect range in highly anisotropic media. Finally, as biological tissue can be modelled using the discrete particle method [8] we have been able to expand our investigation of WFS to tissue-like domains.

References

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